



RESEARCH ARTICLE

Theoretical and numerical investigation of thin double-walled multi-corner single and double-hat columns under quasi-static lateral impact loading

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ABSTRACT

The energy absorption behavior of thin-walled structures under quasi-static lateral impact plays an important role in crashworthy design. This study presents a numerical investigation of thin double-walled multi-corner single-hat (SHDW) and double-hat (DHDW) columns under three-point bending using finite element analysis. Key crashworthiness indicators, such as peak crushing force (P_{peak}), mean crushing force (P_m), energy absorption (EA), specific energy absorption (SEA), and crushing force efficiency (CFE), are evaluated under varying wall thicknesses (0.8–1.2 mm), inner-wall spacings (3–6 mm), and weld pitches (15–30 mm). The numerical results are compared with the super-beam theory, showing that predictive accuracy strongly depends on cross-sectional symmetry. While SHDW exhibits significant discrepancies (~26.7%), DHDW shows excellent agreement (~1.41%). Parametric analysis identifies wall thickness as the dominant factor governing both crushing response and energy absorption, whereas spacing and weld pitch have only minor effects. Notably, DHDW consistently achieves higher SEA while maintaining comparable or improved CFE, indicating a more efficient and stable energy absorption mechanism. These findings highlight the advantages of symmetric double-hat configurations for lightweight crashworthy structures.

1. Introduction

In recent years, enhancing energy absorption while reducing structural weight has become an important objective in crashworthy structural design. Thin-walled structures are widely used in automotive load-bearing components because of their high specific energy absorption and lightweight characteristics. According to the Insurance Institute for Highway Safety (IIHS) [1], side-impact collisions account for approximately 27% of passenger vehicle fatalities in the United States, highlighting the urgent need to improve the lateral crashworthiness of vehicle side structures. Although extensive studies have been conducted on the axial crushing behavior of thin-walled structures, comparatively fewer studies have focused on their bending response under lateral loading.

In early research, square-section single-walled columns were commonly adopted as reference structures to analyze their axial and lateral crushing performance under various loading conditions. A seminal study that laid the theoretical model for explaining the collapse mechanisms of square and rectangular tubes under pure bending was first proposed by Kecman [2] in 1983. Meanwhile, Wierzbicki and Abramow-

icz [3] developed the Super-Folding Element (SE) theory for axial crushing, demonstrating that the bending response of thin-walled tube sections can be derived from their corresponding axial crushing behavior.

Various approaches have been proposed to improve the crashworthiness of thin-walled structures, including multi-cell, foam-filled, and hat-section configurations. Wang et al. [4] developed an analytical model for multi-cell sections, while Sun et al. [5] demonstrated the superior energy absorption of criss-cross tubes. Foam-filled structures have also been shown to enhance bending resistance and energy absorption through improved sectional stability [6–9].

More recently, increasing attention has been devoted to single- and double-hat structures because of their superior bending performance. Duan et al. [10] established an analytical model for top-hat sections under bending, whereas Zhang et al. [11] demonstrated the advantages of irregular double-hat beams through finite element simulations. Bao-Tang Gia et al. [12] investigated the axial crushing behavior of double-walled multi-corner double-hat columns, while Hai Tran et al. [13] and Tam-Huynh Van et al. [14] further showed that double-walled multi-corner structures exhibit excellent bend-

ing energy absorption under lateral loading.

Despite these significant advances, most previous studies have focused on either the axial crushing behavior or the numerical and experimental investigation of bending performance in thin-walled structures. The applicability of existing analytical collapse models to thin double-walled multi-corner single- and double-hat columns under quasi-static lateral loading has not been comprehensively investigated.

To address this gap, the present study combines a generalized analytical model with finite element simulations to investigate the bending behavior and energy absorption characteristics of thin double-walled multi-corner single- and double-hat columns under quasi-static lateral loading. The analytical predictions are compared with the numerical results to evaluate the applicability of the proposed model, while the effects of wall thickness, spot-weld pitch, and inner-wall spacing on crashworthiness are systematically investigated. The overall research methodology is illustrated in Fig. 1.

2. Theoretical background

2.1. Crashworthiness parameters

The crash performance of structures subjected to lateral impact loading is commonly assessed using energy absorption indicators, including total energy absorption (EA), specific energy absorption (SEA), mean crushing force (P_m), and crash force efficiency (CFE). Fig. 2 illustrates the force–displacement curve employed to define these indicators, as expressed in Eqs. (1)–(3).

Energy absorption (EA) represents the capability of a structure to dissipate impact energy via deformation mechanisms. From the force–displacement curve, the total absorbed energy is obtained by integrating the instantaneous crushing force with respect to displacement, as given by

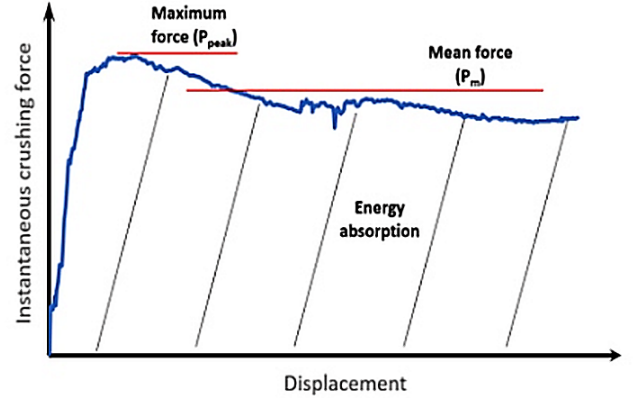


Figure 2. Typical force displacement curve [14]

$$EA = \int_0^{\delta} P(x) dx \quad (1)$$

where $P(x)$ is the instantaneous crushing force; δ is the deformation of structure.

The SEA indicates the energy absorption capacity of thin-walled structures by measuring the amount of energy absorbed per unit mass of the thin-walled beam. A mathematical equation for SEA is

$$SEA = \frac{EA}{m} \quad (2)$$

where m is the total mass of structure.

The CFE indicates the uniformity of the collapse force. This indicator provides insight into how efficiently a structure can absorb energy under compressive loading. A mathematical equation for CFE is

$$CFE = \frac{P_m}{P_{peak}}, \quad (3)$$

in which, P_m is the average crushing force during the whole process and P_{peak} is the maximum force in the process.

For small bending rotation process, Santosa et al. [7] suggested a relation of moment-rotation Fig. 3 as

$$M(\theta) = P(\theta) \frac{L}{4} \quad (4)$$

For small rotation angles, the rotation of beam can be approximated as

$$\theta = \tan^{-1} \frac{2\delta}{L}, \quad (5)$$

where δ_0 is total displacement at central of beam at rotation angle.

2.2. The idealized material model

In crash events, particularly under high-velocity impact, structural components are subjected to severe deformation caused by dynamic loading, which is critical for safety assessment. When plastic deformation is small, referred to as elastic deformation, the material deforms within its elastic limit and can fully recover after unloading. The energy absorbed during this stage is limited due to the linear and reversible stress–strain

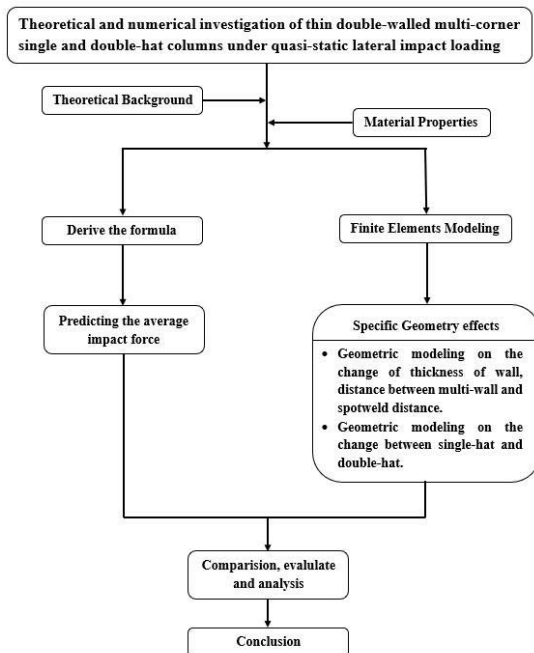


Figure 1. Flow chart of the research for bending behavior of structure

relationship. Once the stress exceeds the yield point, the absorbed energy increases markedly as irreversible changes occur in the material structure. Consequently, the energy absorption capacity of a system during large deformations is significantly greater than that during small deformations. Therefore, in crash analysis, elastic strain is often neglected, and an idealized rigid-plastic material model is commonly adopted.

In this study, the material behavior in the finite element model is described using a rigid power hardening model (curve No. 3 in Fig. 4). Unlike linear hardening, in which the stress-strain relationship is characterized by a constant slope, power hardening provides greater flexibility and can better represent experimental responses of many materials. During the plastic deformation stage, the stress increases following a power-law relationship, which can be expressed mathematically as:

$$\sigma = \begin{cases} \leq \sigma_y & \text{if } \varepsilon = 0 \\ = \sigma_y + K\varepsilon^n & \text{if } 0 \leq \varepsilon \leq \varepsilon_f \end{cases} \quad (6)$$

where K is the hardening modulus; n is the hardening exponent.

2.3. Theoretical collapse model of thin-walled square and rectangular tubes

This section presents the theoretical collapse mechanism of thin-walled square and rectangular tubes under bending loads, building upon the pioneering work of Kecman [2]. The original concepts, including the formation of fixed and mobile plastic hinge lines, local bulging, and gradual flexing during flexural collapse, are restated here to provide a theoretical basis for further analysis.

2.3.1. Collapse process of thin-walled tubes under bending loading

When a thin-walled tubular structure is subjected to bending impact loading, two non-plastically deforming components are formed. These components are defined as stationary hinge lines, which generate two undeformed planes, denoted as GHKNG and EFMLE, as illustrated in Fig. 5. Between these two undeformed planes, the deformation behavior of the hinge lines under bending impact loading can be observed. Based on a detailed analysis of the hinge behavior and deformation modes, the following key observations can be summarized:

- Most of the plastic deformation is concentrated along the stationary plastic hinge lines, namely EF, GH, EB, GB, FC, HC, BC, BA, CJ, GK, NH, EL, and FM.
- Moving plastic hinge lines (rotating hinges) occur along edges GA, EA, KA, and LA. The length of edge BA varies continuously as the rotating hinge evolves, with point A moving from A1 to A3, as shown in Fig. 5(b).
- The angles along hinge lines KL and MN remain constant throughout the deformation process.
- The deformation of the tensile surface (the lower face of the model) contains two moving plastic hinge lines, NK and ML, and is approximated by the intersection between the stationary plane and the cylindrical surface ID.

- Edges GA, EA, BA, HC, CF, and CJ act as rotating hinges where membrane deformation develops; this deformation contributes to maintaining the kinetic energy of the hinges.
- The rolling radius r varies slightly along GA (measured on sections perpendicular to GA) and its estimated value is approximately proportional to the hinge length, $r = (0.03\text{--}0.05)KL$. Larger values correspond to thicker tubes with equivalent cross-sections. The radius decreases during collapse, progressively restricting rotational deformation until the mechanism becomes fully “locked,” initiating a new collapse stage.
- The rotation radius along edges KA, LA, MJ, and NJ varies significantly, ranging from infinity at vertices K, L, M, and N to the radius r defined above. The deformation mode along these edges is not clearly defined.

Four distinct phases in the hinge collapse process are identified:

- Initial phase, during which local bulging develops at the tube edges, while rotational deformation has not yet occurred with a “bulge” in the webs (A_1 in Fig. 5b) starting approx
- Second phase, characterized by the onset of rotational deformation along hinge lines GA, KA, EA, and LA, and the cross-section shape as in Fig. 5(b) (A_2)
- Third phase, in which rotational deformation becomes “locked,” leading to the formation of new plastic hinge lines, with the cross-section taking the shape B_3A_3TD in Fig. 5(b) and with new bending lines AT and AL and their symmetric counterparts.
- Final phase, initiated when the two bulged regions on the compressive surface come into contact; at this stage, the original hinge lines are fully locked and adjacent new hinge lines develop. The compressive surface buckles outward, whereas the side walls buckle inward. These deformation patterns are not clearly separable and are therefore treated as a mixed phase.

2.3.2. Theoretical collapse mechanism

The theoretical model of the hinge collapse mechanism is established based on the second phase of the collapse process. The assumptions adopted in this theoretical framework include:

- Deformation of the surfaces is confined to the plastic hinge lines.
- The surfaces are assumed to be incompressible and inextensible.
- Structural continuity is preserved both across the cross-section and along the longitudinal direction.
- The hinge length $KL = 2h$ and the rotation radius r are illustrated in Fig. 6. The parameters h and r are determined using the minimum moment method; therefore, a new model is developed, as shown in Fig. 6.

Based on the model shown in Fig. 6, the following quantities can be determined:

$$x_B = h; y_B = b \cos \rho - \sqrt{b \sin \rho (2h - b \sin \rho)}; z_B = 0 \quad (7)$$

$$b = z_A + \sqrt{y_A^2 + z_A^2} \text{ where } y_A = y_B; \quad (8)$$

where $\rho = \frac{\theta}{2}$, with θ denoting the rotation angle of the hinge induced by bending impact.

From Eqs. (7) and (8), it follows that:

$$z_A = b \sin^2 \rho - h \sin \rho + \sqrt{b \sin \rho (2h - b \sin \rho)} \cos \rho \quad (9)$$

Under the assumptions that the tube walls are inextensible and the rotation radius is sufficiently small, the following relation holds:

$$h^2 = [h - (b - z_A) \sin \rho]^2 + [y_A - (b - z_A) \cos \rho]^2 + z_A^2 \quad (10)$$

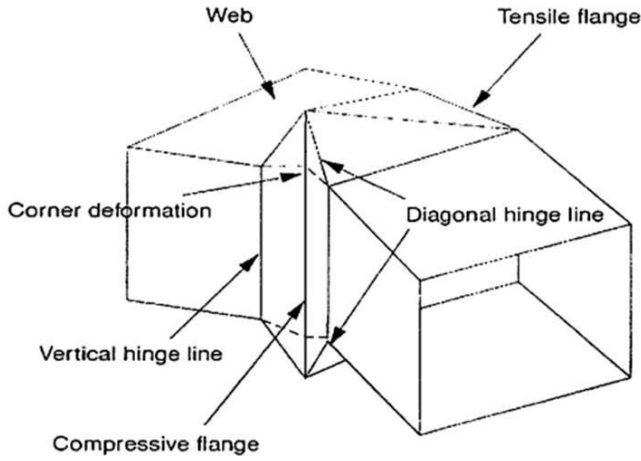


Figure 3. Bending collapse mode [15]

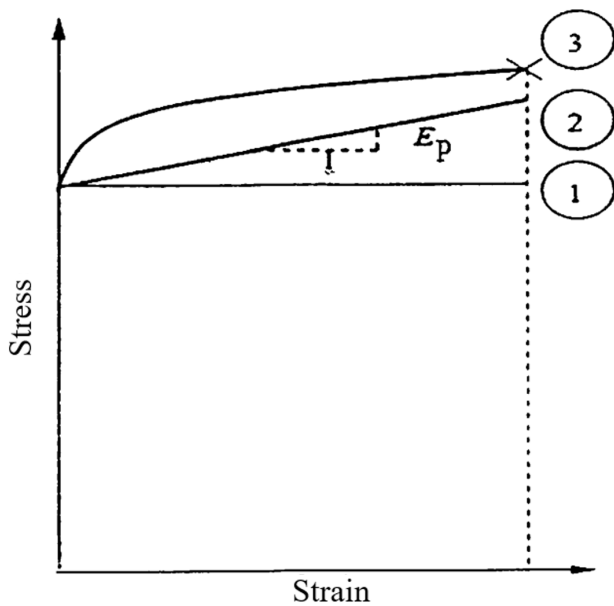


Figure 4. Idealized stress-strain curves of rigid materials [15]: (1) perfectly plastic; (2) linear hardening; (3) power hardening

By substituting Eqs. (7) and (8) into Eq. (10), the following expression is obtained:

$$2h = a \text{ with } a \leq b, \quad \text{or } 2h = b \text{ with } a \geq b \quad (11)$$

The collapse mechanism is locked when the two bulged regions of the compression face come into contact. The corresponding locking rotation angle is

$$\theta_j = 2 \sin^{-1} \left(\frac{h - 0.5t}{b} \right). \quad (12)$$

2.4. Theoretical analysis of bending behavior

2.4.1. Kinematics of the super beam element

Since a tube subjected to bending possesses only one plane of symmetry, a half-tube model, as shown in Fig. 7, is adopted for the theoretical bending analysis. This model, referred to as the special beam element, or super beam element (SB), was originally proposed by Wierzbicki in 1994 [7]. The model consists of three planes: the upper plane, which is directly subjected to the external load and acts as the compression face; the lower plane, which serves as the tension face; and the side plane, where a transition from compression to tension occurs. For simplicity, a single SB element can be regarded as comprising two SE elements, where the upper element under compression is denoted as SE₁, while the element containing the tension face is denoted as SE₂.

Specifically, SE1 refers to the corner sub-element associated with the compression side of the SB element, that is, the part containing the compression flange and the adjacent web portion. This sub-element deforms in a manner analogous to a super-folding element under axial crushing and therefore dissipates energy mainly through corner deformation, rolling deformation, and bending along the stationary hinge lines in the compressed flange and web.

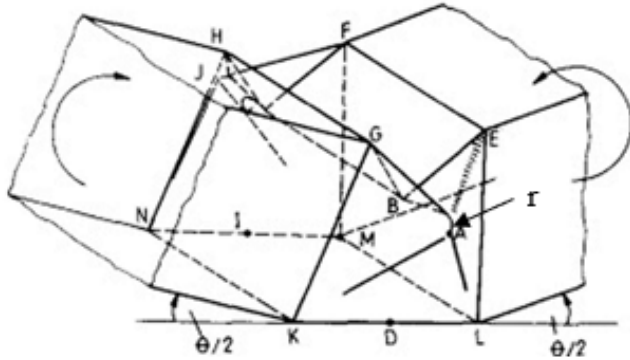
SE2 denotes the corner sub-element associated with the tension side of the SB element, that is, the part containing the tensile flange and the adjacent web portion. Although SE2 is referred to as the tensile element, part of this region may still experience compression in the web, while the tensile flange primarily contributes through stretching and associated hinge deformation.

As shown in Fig. 7, the side face of the SB element consists of five triangular segments that undergo relative rotational motion characterized by the angles α , β , and γ . Among these, β and γ can be expressed in terms of the characteristic rotation angle α , which can further be related to the vertical displacement δ . However, for easier observation, the specific positions of the angles can be seen in Fig. 6b. The geometric dimensions of the side face of the SB element are illustrated in Fig. 8. During the collapse process, the position of the neutral axis rapidly shifts toward the tension face; therefore, the parameter η remains much smaller than b for most of the deformation history.

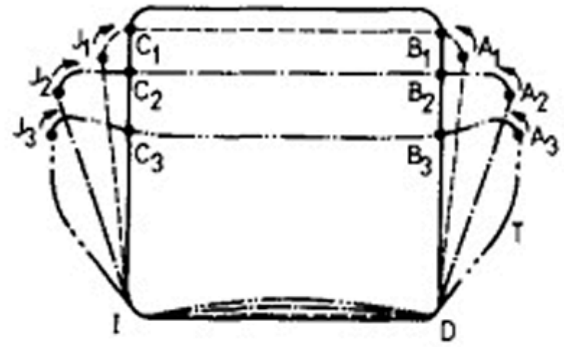
2.4.2. Global equilibrium of the Super beam Element

The energy absorption of the Superbeam Element in bending theory is determined using the principle of virtual work.

$$\dot{E}_{\text{ext}} = \dot{E}_{\text{int}} \quad (13)$$

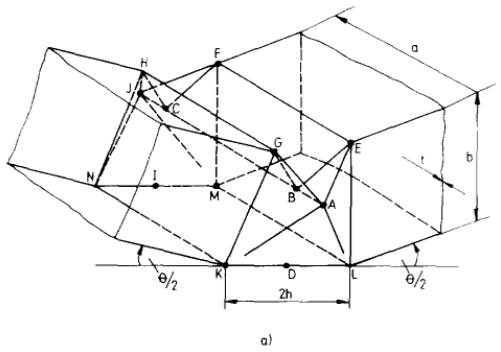


a)

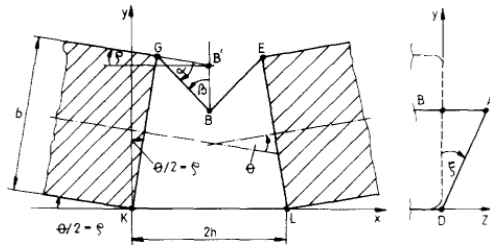


b)

Figure 5. Theoretical collapse model (a) and collapse stages (b) [2]



a)



b)

Figure 6. Theoretical collapse model (a) and longitudinal cross-section (b) [2]

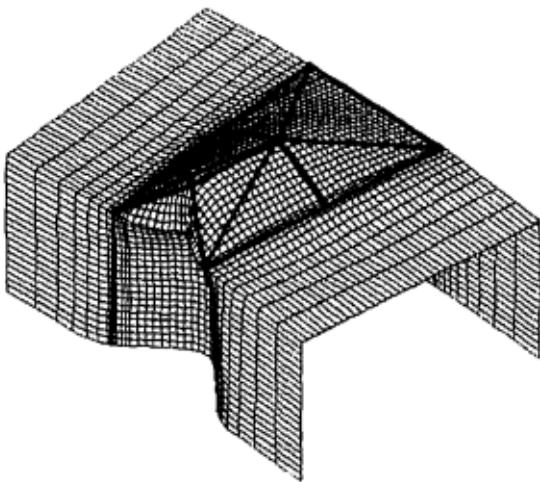


Figure 7. Superbeam Element model in FEM [7]

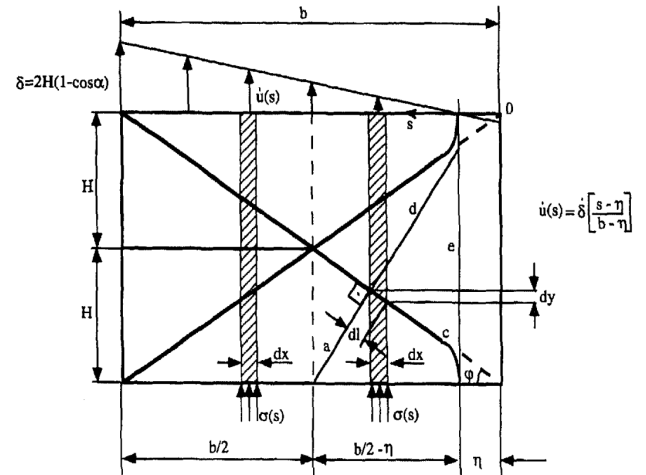


Figure 8. The geometric dimensions of the side face of the SB element [7]

The internal energy dissipation of the SB element involves more than three additional components relative to the Superfolding element.

$$P\delta = \dot{E}_{cor} + \dot{E}_{rol} + \frac{1}{2}\dot{E}_b + \dot{E}_{w1} + \dot{E}_{w2} + \dot{E}_{ten} \quad (14)$$

where E_{cor} is the rate of internal energy dissipation due to corner extensional deformation, namely the plastic work associated with stretching of the corner region during progressive folding. E_{rol} is the rate of internal energy dissipation due to rolling deformation in the corner zone, the plastic work required for the moving plastic hinge to roll with finite radius during fold development. E_b is the rate of internal energy dissipation associated with plastic bending along the stationary hinge lines, particularly those forming in the flange and web portions of the SB mechanism. $\dot{E}_{w1}$ and $\dot{E}_{w2}$ are the rates of energy dissipation in the SE_1 and SE_2 elements, respectively, and $\dot{E}_{ten}$ denotes the rate of energy dissipation in the tension flange. The contribution $\dot{E}_b$ is halved because one half of the energy has already been dissipated by the SE_1 element. The rate of energy dissipation $\dot{E}_{w1}$ can be expressed as

$$\dot{E}_{w1} = 2 \sum_i m_p \psi_o l_i \quad (15)$$

By combining the three additional energy components with the three energy components in the axial crushing theory derived by Wierzbicki (1994) [7], the instantaneous crushing force of the eccentrically loaded column is obtained as follows:

$$P(\alpha) \sin \alpha = m_p \left\{ \begin{aligned} &8I_1 \frac{r}{t} + 2I_2(\alpha) \frac{H}{r} + 2\frac{b}{h} \\ &+ \frac{\sqrt{2}}{2} \frac{b}{H} \left[1 + \left(\frac{H}{\frac{b}{2} - \eta} \right)^2 \right] \\ &+ 4\frac{b}{t} \frac{\eta}{b-\eta} \sin \alpha + \left(2 - \sqrt{2} \right) \frac{H}{b} \end{aligned} \right\} \quad (16)$$

In Eq. (16), $P(\alpha)$ depends on three unknown parameters, namely r , H , and η . The parameters r and H are determined by minimizing the mean crushing force, as discussed in the subsequent section, following the same approach as in the axial crushing theory. The parameter η is obtained from the force equilibrium condition; however, its determination is beyond the scope of the present study.

2.4.3. Determination of bending wavelength

By integrating Eq. (16) with respect to α from 0 to $\frac{\pi}{2}$, the mean crushing force is obtained as follows:

$$P_m = m_p \left\{ \begin{aligned} &4.64 \frac{r}{t} + 2.3 \frac{H}{r} + \pi \frac{b}{h} \\ &+ \int_0^{\pi/2} \left[\frac{\sqrt{2}}{2} \frac{b}{H} \left[1 + \left(\frac{H}{\frac{b}{2} - \eta} \right)^2 \right] + \left(2 - \sqrt{2} \right) \frac{H}{b} \right] d\alpha \\ &+ 4\frac{b}{t} \int_0^{\pi/2} \left(\frac{\eta}{b-\eta} \sin \alpha \right) d\alpha \end{aligned} \right\} \quad (17)$$

The first three terms can be integrated based on the bending theory, while the integration of the last two terms cannot be readily performed because η is still an unknown function. However, by applying the integral continuity condition, Eq. (17) can be rewritten as:

$$P_m = m_p \left\{ 4.64 \frac{r}{t} + 2.3 \frac{H}{r} + \frac{3\pi}{2} \frac{b}{h} + 4\frac{b}{t} \int_0^{\pi/2} \frac{\eta}{b-\eta} \sin \alpha d\alpha + \pi \left(2 - \sqrt{2} \right) \frac{H}{b} \right\} \quad (18)$$

The remaining integral term does not contribute to the minimization procedure, as it does not depend on the parameters r and H . Consequently, the contribution from the tension face, which accounts for less than 2% of the mean crushing force, can be neglected [7]. Under these assumptions, the analytical minimization of the mean crushing force P_m with respect to r and H can be carried out as

$$\frac{\partial P_m}{\partial H} = 0 \quad (19)$$

and

$$\frac{\partial P_m}{\partial r} = 0 \quad (20)$$

The solutions of the above Eqs. (19), (20) are

$$H = 1.276b \frac{2}{3} t^{\frac{1}{3}} \quad (21)$$

and

$$r = 0.7955b \frac{1}{3} t^{\frac{2}{3}} \quad (22)$$

2.5. Generalization of the theoretical prediction of the mean crushing force for arbitrary multi-corner columns

As shown above, the mean crushing force of an arbitrary thin-walled multi-corner tube can be predicted by subdividing the tube cross-section into a set of special beam (SB) elements. This procedure simplifies the analysis while leaving the bending element length and the hinge rolling radius unchanged. The generalized expression for the mean crushing force of an arbitrary multi-corner thin-walled tube is derived by extending the super-beam (SB) bending model proposed by Wierzbicki et al. [7]. The mean crushing force of an arbitrary multi-corner tube is therefore given by:

$$P_m = 2 \sum_{i=1}^n N_i M_o \left(4.64 \frac{r}{t} + 2.3 \frac{H}{r} + \frac{3\pi}{2} \frac{b_i}{H} + 4\frac{b}{t} \int_0^{\pi/2} \frac{\eta}{b-\eta} \sin \alpha d\alpha + \pi \left(2 - \sqrt{2} \right) \frac{H}{b} \right) \quad (23)$$

where: n : denotes the number of types of SB elements in the cross-section of the multi-corner tube; N_i : is the number of SB elements of type i in one half of the cross-section; H : is half of the bending element length; b_i : represents the mean side length of the i -th SB element; r : is the hinge rotation (rolling) radius; t : denotes the tube wall thickness; η : is the position of the neutral axis along the side wall of the tube.

M_o : is the fully plastic bending moment per unit length of a plastic hinge.

In Eq. (23), the term containing the integral is eliminated since it does not depend on the two parameters to be determined, namely the bending element length and the hinge rotation radius. Moreover, the influence of the neutral axis position η on the mean crushing force is negligible. The last term in Eq. (23) is also neglected because its contribution to the mean crushing force is less than 2%. Consequently, the expression for the mean crushing force of an arbitrary multi-corner tube reduces to:

$$P_m = \sum_{i=1}^n N_i M_o \left\{ 4.64 \frac{r}{t} + 2.3 \frac{H}{r} + \frac{3\pi}{2} \frac{b_i}{H} \right\} \quad (24)$$

The derivative of the P_m with respect to the half folding wavelength H , and the rolling radius r is given by

$$\frac{\partial P_m}{\partial H} = \sum_{i=1}^n N_i M_o \left(2.3 \frac{1}{r} - \frac{3\pi}{2} \frac{b_i}{H^2} \right) \quad (25)$$

and

$$\frac{\partial P_m}{\partial r} = \sum_{i=1}^n N_i M_o \left(4.64 \frac{1}{t} - 2.3 \frac{H}{r^2} \right). \quad (26)$$

To determine the optimal values of the half folding wavelength H and the rolling radius r , the resulting system of two equations is solved:

$$\frac{\partial P_m}{\partial H} = \sum_{i=1}^n N_i M_o \left(2.3 \frac{1}{r} - \frac{3\pi}{2} \frac{b_i}{H^2} \right) = 0 \quad (27)$$

$$\frac{\partial P_m}{\partial r} = \sum_{i=1}^n N_i M_o \left(4.64 \frac{1}{t} - 2.3 \frac{H}{r^2} \right) = 0 \quad (28)$$

From Eqs. (27) and (28), the following relation is obtained:

$$4.64 \frac{1}{t} - 2.3 \frac{H}{r^2} = 0 \Leftrightarrow H = \frac{4.64r^2}{2.3t} = 2.017 \frac{r^2}{t} \quad (29)$$

Substituting Eq. (29) into Eq. (27) yields the corresponding expression:

$$\begin{aligned} \sum_{i=1}^n N_i M_o \left[2.3 \frac{1}{r} - \frac{3\pi}{2} b_i \left(\frac{t}{2.017r^2} \right)^2 \right] &= 0 \\ \Leftrightarrow \sum_{i=1}^n N_i [2.3r^2 - 0.3686\pi b_i t^2] &= 0 \\ \Leftrightarrow r^3 = 0.16\pi t^2 \frac{\sum_{i=1}^n N_i b_i}{\sum_{i=1}^n N_i} \\ \Leftrightarrow r = 0.7955t^{2/3} \left(\frac{C_{1/4}}{N} \right)^{1/3} \end{aligned} \quad (30)$$

By further substituting Eq. (30) into Eq. (29), a general formula for the half folding wavelength H is derived as below.

$$\begin{aligned} H &= 2.017 \left[0.7955t^{2/3} \left(\frac{C_{1/4}}{N} \right)^{1/3} \right]^2 \frac{1}{t} \\ \Rightarrow H &= 1.276t^{1/3} \left(\frac{C_{1/4}}{N} \right)^{2/3} \end{aligned} \quad (31)$$

Eq. (31) thus represents a generalized expression applicable to thin-walled multi-corner tubes with an arbitrary number of corners, consistent with the formulation proposed by Wierzbicki and Abramowicz 1994 [7] for square and rectangular tubes. For square and rectangular tubes, the number of SB elements in one half of the cross-section, N , equals unity, and one quarter of the cross-sectional perimeter, $C_{1/4}$, corresponds to the mean edge length.

3. Material and Numerical modeling

3.1. Material properties

For the numerical simulations, the thin double-walled structures are assumed to be made of hot-rolled St37-HR steel. The material behavior was characterized by quasi-static tensile tests at a strain rate of 0.016 s^{-1} , with experimental data taken from Tam-Huynh Van et al. [14]. The true stress–true strain curve (Fig. 9) was used to define the plastic material model in ABAQUS and determine the rigid power hardening parameters (σ_y , K , and n). The corresponding material properties are summarized in Table 1.

Table 1. Material properties

Property	HR-St37
Young modulus, E (MPa)	206×10^3
Density, ρ (kg/mm ³)	7.85×10^{-6}
Poisson ration, ν	0.3
Yield strength, σ_y (MPa)	242
Ultimate strength, σ_u (MPa)	347
Hardening exponent, n	0.26

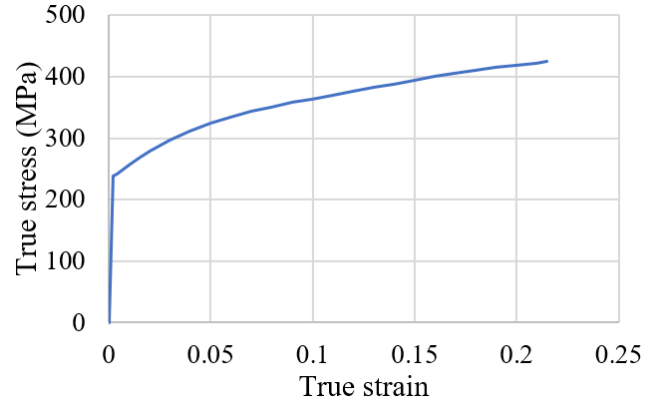


Figure 9. True stress strain curve for HR-St37 used in this work

3.2. Finite element modeling

A finite element model is established in ABAQUS/Explicit to predict and analyze the bending characteristics of thin double-walled multi-corner single and double-hat columns under quasi-static three-point bending tests. In the finite element model, the impact hammer and the two supports are modeled as discrete rigid cylindrical bodies. Each rigid body has a diameter of 30 mm and a length of 90 mm, and the distance between the centers of the two supports is 400 mm. The impacted structure is modeled using S4R shell elements, i.e., four-node quadrilateral shell elements with reduced integration, which are suitable for three-dimensional nonlinear analyses involving large deformations of thin-walled structures. To balance computational efficiency and accuracy, a non-uniform mesh is adopted, with a refined mesh of $2 \times 2 \text{ mm}^2$ in the impact region is employed in the central impact region and a coarser mesh of $20 \times 2 \text{ mm}^2$ elsewhere. This meshing strategy is consistent with previous studies [14] and was further verified in the present work by comparison with a uniformly refined $2 \times 2 \text{ mm}^2$ mesh, showing excellent agreement in the predicted force-displacement response (Fig. 10). The finite element model of the structure subjected to lateral impact loading is illustrated in Fig. 11.

The impact velocity is directly prescribed to the impact hammer in the form of a velocity boundary condition under a quasi-static loading scheme to reproduce the experimental three-point bending test. Specifically, the hammer velocity is smoothly ramped from 0 to 0.1 m/s during the initial 50 ms using a smooth ramp function. After the acceleration phase, the velocity is maintained constant at 0.1 m/s throughout the impact process. The velocity–time history applied to the impact hammer in the numerical simulation is shown in Fig. 12.

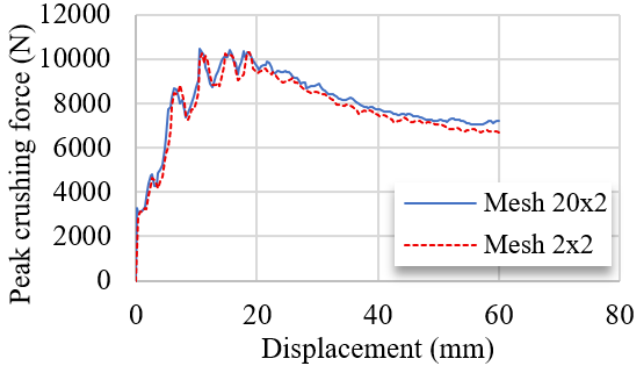


Figure 10. FEM results for the two mesh sizes: 20x2 and 2x2

The spot welds along the flanges were modeled using *Structural Distributed Coupling* together with *Multi-Point Constraint* (MPC) beam connections in ABAQUS. A reference point was defined at the center of each spot weld, and the surrounding shell nodes were coupled to the reference point to represent the spot-weld connection between the inner and outer walls. The spot-weld modeling technique is illustrated in Fig. 14.

The interactions between the impact hammer and the structure, as well as between the tube and the two supports, were modeled using the “*General Contact*” algorithm in ABAQUS/Explicit. “*Tangential behavior*” was defined using a *static–kinetic exponential decay* friction model, with a static friction coefficient of $\mu_s = 0.74$ and a kinetic friction coefficient of $\mu_k = 0.57$, adopted from the experimental results reported by Annisa et al. [16]. The “*Normal behavior*” was defined as *hard contact*.

3.3. The thin double-walled multi-corner single and double-hat columns geometry

The cross-sectional geometries of the thin double-walled multi-corner single and double-hat columns, hereafter referred to as SHDW and DHDW, respectively, are shown in Fig. 13. The thin double-walled multi-corner structure is generated by adding an inner wall to the single-walled configuration shown in Fig. 13a. Similarly, for the double-hat configuration, the bottom plate is removed and two SHDW are joined to construct the DHDW in Fig. 13c. This study will investigate the bending characteristics of SHDW and DHDW under different wall thicknesses, inner–outer wall spacings, and spot-weld pitches.

For the analytical prediction, the multi-corner cross-sections shown in Fig. 13 are represented by a series of Super Beam (SB) elements according to the generalized formulation presented in Section 2.5. The analytical model is an idealized mechanics-based representation of the dominant plastic collapse mechanism and does not explicitly reproduce all geometric and boundary details of the numerical model. Nevertheless, the analytical and numerical models share the same overall three-point bending configuration and dominant collapse mechanism, allowing their global mean crushing forces to be compared.

Table 2. Geometrical detail in Fig. 13

Outer wall		Inner wall				
Edge	Width (mm)	Edge	Width (mm)			
			$D = 3$	$D = 4$	$D = 5$	$D = 6$
o_1	30	i_1	27	26	25	24
o_2	20	i_2	14	12	10	8
o_3	10	i_3	10	10	10	10
o_4	20	i_4	26	28	30	32

The spacing between the inner and outer walls plays a critical role in the energy absorption and stability of SHDW and DHDW structures. An excessively small spacing limits the available deformation space between the two walls and is impractical for manufacturing, whereas an overly large spacing may reduce the global stiffness, leading to instability or diminished load resistance. To identify an optimal configuration, four wall spacings ($D = 3\text{--}6$ mm) are examined and evaluated in terms of *SEA*, *CFE* and structural stability.

The detailed cross-sectional dimensions of the inner and outer walls corresponding to the four spacing configurations are summarized in Table 2. All conical tube structures investigated in this study have an identical length of $L = 470$ mm.

The inner and outer walls are joined by spot welding along the flange regions. In the baseline numerical model, a constant weld pitch of 20 mm is adopted as displayed in Fig. 14. Furthermore, the effect of spot-weld spacing on energy absorption and deformation behavior is examined by varying the weld pitch to $W = 15, 20, 25$, and 30 mm.

Table 3 summarizes the geometric parameters and mass of the multi-corner tubular specimens investigated in this study, including the wall thickness t (mm), the spacing between the inner and outer walls D (mm), the spot-weld pitch W (mm), and the corresponding mass m (kg). For clarity and consistency throughout the analysis, each specimen is identified using a unified designation system, as described below.

The symbol “ T ” followed by a numerical value denotes the wall thickness of the structure:

- T08: $t = 0.8$ mm
- T10: $t = 1.0$ mm
- T12: $t = 1.2$ mm

The symbol “ D ” followed by a numeral indicates the spacing between the inner and outer walls:

- D3: $D = 3$ mm
- D4: $D = 4$ mm
- D5: $D = 5$ mm
- D6: $D = 6$ mm

The symbol “ W ” followed by a numerical value directly represents the spot-weld pitch in millimeters:

- W15: $W = 15$ mm
- W20: $W = 20$ mm
- W25: $W = 25$ mm
- W30: $W = 30$ mm

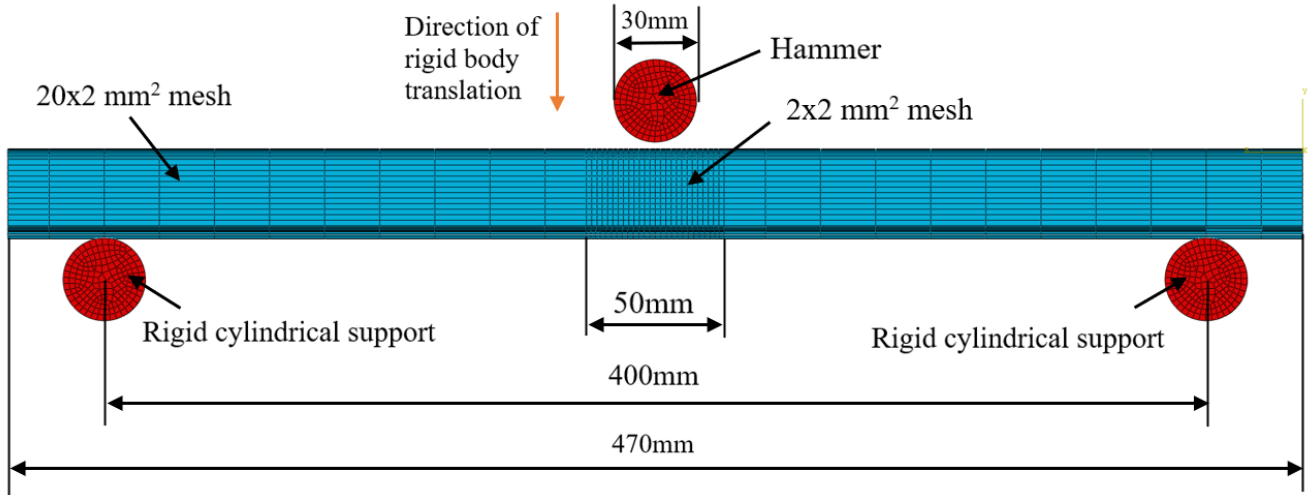


Figure 11. Finite element model of structures subjected to lateral impact loading

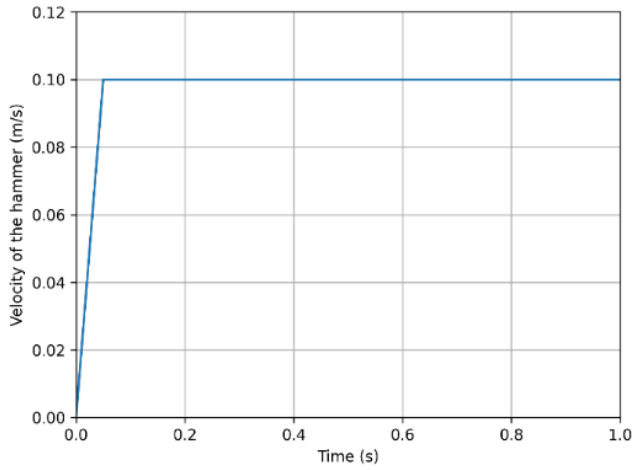


Figure 12. Velocity–time history of the impact hammer

4. Result and Discussions

4.1. Theoretical prediction of mean crushing force and numerical validation

a) SHDW – T10D4W20

For clarity and conciseness, the theoretical formulation is presented in detail only for the representative SHDW and DHDW specimens with configuration T10D4W20. The theoretical results for the remaining cases are summarized directly in Table 8.

The material parameters used in the theoretical calculations are identical to those listed in Table 1. The material flow stress adopted in the collapse model is defined by the following equation:

$$\sigma_o = \sqrt[3]{\frac{2 \times \sigma_y \times \sigma_u^2}{(n+1)^2(n+2)}} \quad (32)$$

$$\sigma_o = \sqrt[3]{\frac{2 \times 242 \times 347^2}{(0.26+1)^2(0.26+2)}} = 253.25 \text{ MPa}$$

The mean crushing force of an arbitrary multi-corner tube is determined based on the method of discretizing the tube

into special beam elements. For any tubular structure, when three fundamental components, including the top edge, the side edge, and the bottom edge, are assembled, a special beam element can be formed. The average side length of this element, denoted as SB, is determined through a general expression that is consistent with the geometric characteristics of each type of tube.

$$\bar{b} = \frac{b_t + 2 \times b_s + b_b}{4} \quad (33)$$

where:

b_t : length of the top edge,

b_s : length of the side edge,

b_b : length of the bottom edge.

For an arbitrary tubular cross-section, the quarter perimeter of the cross-section can be expressed as:

$$C_{1/4} = \sum_{i=1}^n N_i b_i = N_1 b_1 + N_2 b_2 + \dots + N_n b_n \quad (34)$$

where:

N : number of identical SB elements,

b : mean edge length determined according to Eq. (33).

First, Eq. (33) is employed to determine the mean length of the super beam (SB) elements for both the outer and inner walls, as listed in Table 4. The locations and number of these elements are illustrated in Fig. 15.

The calculated results are summarized in Table 5 using Eqs. (24), (30), (31) and (34) presented in Section 2.5 and Section 4.1.

b) DHDW – T10D4W20

For the DHDW configuration, the theoretical evaluation is carried out following the same analytical procedure as that applied to the SHDW structure. The mean lengths of the super beam (SB) elements corresponding to the outer and inner walls are determined accordingly and summarized in Table 6. The distribution and number of these SB elements are schematically illustrated in Fig. 16.

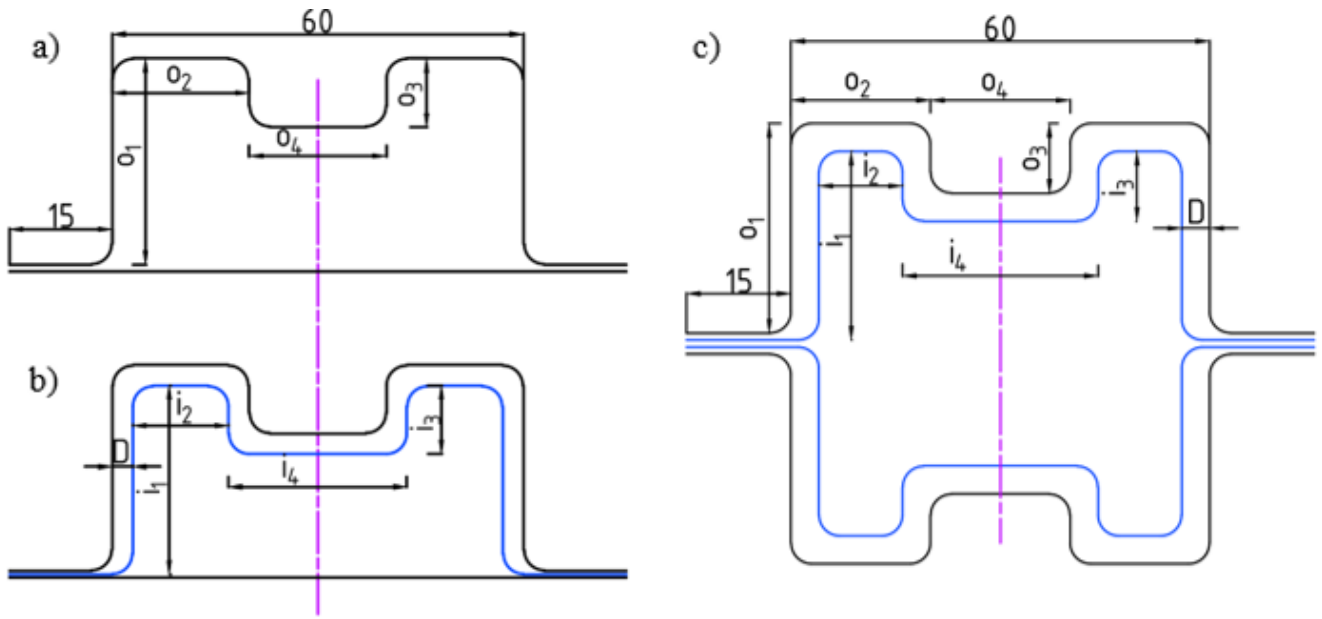


Figure 13. Cross-sectional geometry of structures: (a) single-hat single-walled; (b) single-hat double-walled; (c) double-hat double-walled

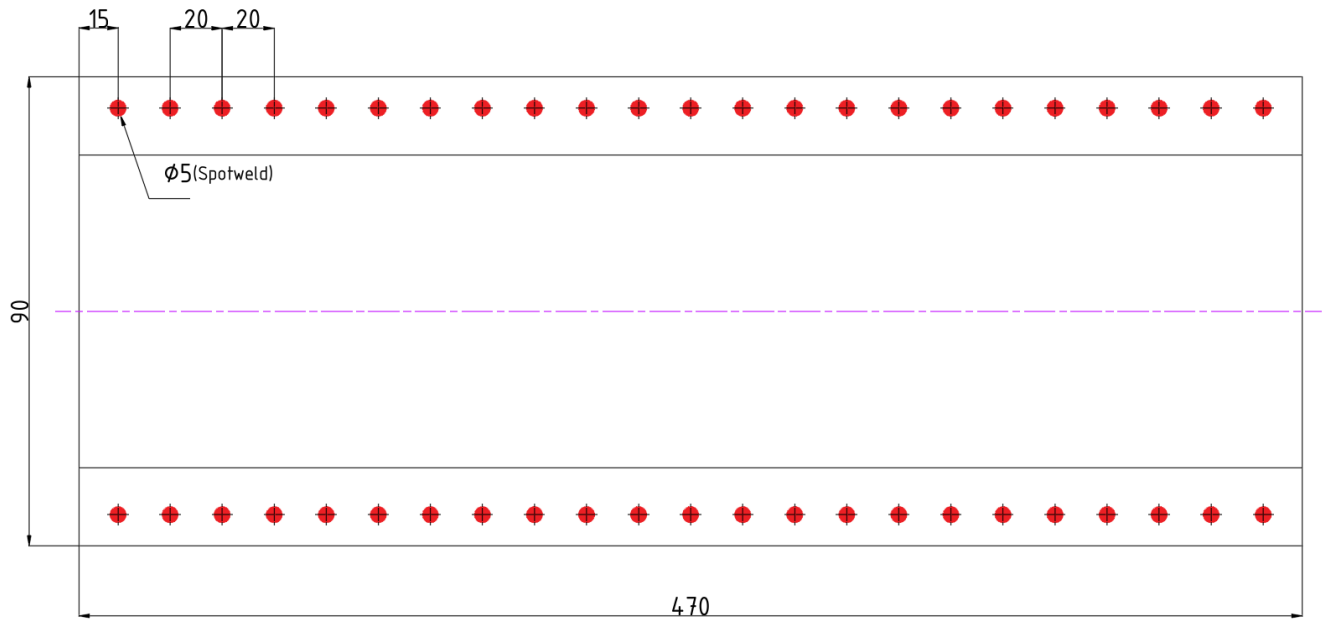


Figure 14. The length of the structure and the spot-weld layout along the flanges

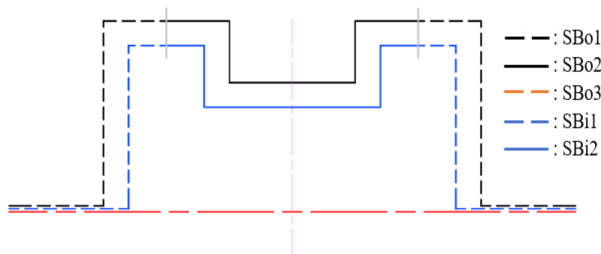


Figure 15. Super beam elements for the inner and outer walls of the SHDW cross-section

Based on the obtained SB parameters, the mean impact force of the DHDW structure is subsequently calculated using the theoretical formulations introduced in **Section 2.5**, and the resulting values are reported in **Table 7**.

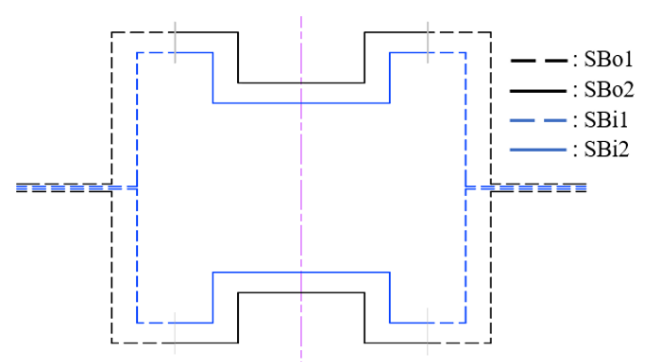


Figure 16. Super beam elements for the inner and outer walls of the DHDW cross-section

Table 3. Geometric parameters and mass of the investigated multi-corner tubular specimens

Code	Thickness (mm)	Mass (kg)
SHDW-T08D3W20	0.8	1.1904
SHDW-T08D4W20		1.1845
SHDW-T08D5W20		1.1786
SHDW-T08D6W20		1.1727
SHDW-T10D3W20	1	1.4881
SHDW-T10D4W20		1.4807
SHDW-T10D5W20		1.4733
SHDW-T10D6W20		1.4659
SHDW-T12D3W20	1.2	1.7857
SHDW-T12D4W20		1.7768
SHDW-T12D5W20		1.768
SHDW-T12D6W20		1.7591
DHDW-T08D3W20	0.8	1.8496
DHDW-T08D4W20		1.8378
DHDW-T08D5W20		1.826
DHDW-T08D6W20		1.8142
DHDW-T10D3W20	1	2.312
DHDW-T10D4W20		2.2972
DHDW-T10D5W20		2.2825
DHDW-T10D6W20		2.2677
DHDW-T12D3W20	1.2	2.7744
DHDW-T12D4W20		2.7567
DHDW-T12D5W20		2.739
DHDW-T12D6W20		2.7213

Table 4. Mean edge length of elements in single-hat double-walled columns

Element	Number of elements	Mean edge length (mm)
SBo1	1	$\frac{20 + 2 \times 30 + 30}{4} = 27.5$
SBo2	1	$\frac{20 + 10 \times 2 + 20}{4} = 15$
SBo3	1	$\frac{90}{4} = 22.5$
SBi1	1	$\frac{12 + 2 \times 26 + 36}{4} = 25.5$
SBi2	1	$\frac{12 + 2 \times 14 + 10 \times 2}{4} = 15$

4.2. Validation of finite element model

To validate the numerical model of structures, the simulation results are evaluated against the analytical predictions obtained from the theoretical formulations introduced in **Section 2** and the representative cases presented in **Section 4.1**.

Table 5. Calculated mean crushing force of SHDW

Variable	Outer wall	Inner wall
Thickness, t (mm)	1	1
The half folding wavelength, H (mm)	9.917	9.48
Rolling radius, r (mm)	2.218	2.168
P_m (N)	$P_m^{SBo1} = 2129.58$ $P_m^{SBo2} = 1753.71$ $P_m^{SBo3} = 1979.23$ $\sum P_m^{SBo} = 5862.52$ $\sum P_m = 9683.77$ (N)	$P_m^{SBo1} = 2075.77$ $P_m^{SBo2} = 1745.48$ – $\sum P_m^{SBo} = 3821.25$

Table 6. Mean edge length of elements in double-hat double-walled columns

Element	Number of elements	Mean edge length (mm)
SBo1	2	$\frac{20 + 2 \times 30 + 30}{4} = 27.5$
SBo2	2	$\frac{20 + 10 \times 2 + 20}{4} = 15$
SBi1	2	$\frac{12 + 2 \times 26 + 36}{4} = 25.5$
SBi2	2	$\frac{12 + 2 \times 14 + 20}{4} = 15$

Table 7. Calculated mean crushing force of DHDW

Variable	Outer wall	Inner wall
Thickness, t (mm)	1	1
The half folding wavelength, H (mm)	9.789	9.48
Rolling radius, r (mm)	2.203	2.168
P_m (N)	$P_m^{SBo1} = 2131.95$ $P_m^{SBo2} = 1751.18$ $\sum P_m^{SBo} = 7766.26$ $\sum P_m = 15408.76$ (N)	$P_m^{SBo1} = 2075.77$ $P_m^{SBo2} = 1745.48$ $\sum P_m^{SBo} = 7642.5$

a) SHDW with changes in thickness and distance between two walls

Table 8 shows a noticeable discrepancy between the theoretical and numerical mean crushing forces for the SHDW configuration. The mean difference is approximately 26.71%, with values ranging from 22.19% to 31.06%. This difference may be attributed to the assumptions underlying the analytical model, which was originally developed for symmetric single-cone structures with ideal axisymmetric folding.

According to Tomasz Wierzbicki [7], the neutral axis is determined based on equilibrium and stress continuity under the assumption of symmetric stiffness distribution. While this assumption is appropriate for symmetric sections, the introduction of an external plate in the SHDW configuration leads to a non-uniform stiffness distribution across the cross-section. As a result, the position of the neutral axis may evolve during the deformation process.

Table 8. Comparison of mean crushing force (P_m) between numerical simulation and theoretical prediction for SHDW specimens ($W = 20$ mm)

Specimen	P_m (N) (FEM)	P_m (N) (Theory)	Difference (%)
T08D3W20	5179.04	6686.97	29.12
T08D4W20	5144.80	6676.18	29.77
T08D5W20	5107.20	6665.29	30.51
T08D6W20	5077.14	6654.31	31.06
T10D3W20	7777.97	9699.43	24.70
T10D4W20	7753.83	9683.77	24.89
T10D5W20	7647.03	9667.98	26.43
T10D6W20	7534.26	9652.06	28.11
T12D3W20	10756.63	13143.62	22.19
T12D4W20	10699.49	13122.40	22.65
T12D5W20	10496.18	13101.00	24.82
T12D6W20	10354.17	13079.43	26.32

Table 9. Comparison of mean crushing force (P_m) between numerical simulation and theoretical prediction for DHDW specimens ($W = 20$ mm)

Specimen	P_m (N) (FEM)	P_m (N) (Theory)	Difference (%)
T08D3W20	10477.40	10644.69	1.60
T08D4W20	10567.61	10623.09	0.52
T08D5W20	10610.96	10601.32	0.09
T08D6W20	10723.13	10579.36	1.34
T10D3W20	15391.63	15440.08	0.31
T10D4W20	15674.25	15408.76	1.69
T10D5W20	15627.48	15377.18	1.60
T10D6W20	15710.86	15345.33	2.33
T12D3W20	20946.79	20922.73	0.11
T12D4W20	21510.73	20880.28	2.93
T12D5W20	21127.07	20837.49	1.37
T12D6W20	21427.91	20794.33	2.96

In addition, the asymmetric stiffness distribution may introduce additional bending effects and local interactions between the outer plate and inner conical walls. These factors can influence the load transfer mechanism and energy dissipation behavior. Since such effects are not explicitly considered in the analytical formulation, the observed differences between theoretical and numerical results are expected. This observation highlights the sensitivity of the analytical prediction to underlying geometric and mechanical assumptions.

b) DHDW with changes in thickness and distance between two walls

Table 9 shows good agreement between the numerical and theoretical mean crushing forces for the DHDW configuration. The average difference is approximately 1.41%, ranging from 0.09% to 2.96%, which is significantly smaller than that observed for the SHDW case.

Table 10. Effect of wall thickness and inner-wall distance on the numerical crashworthiness performance of SHDW ($W = 20$ mm)

Specimen	P_{peak} (N)	SEA (J/kg)	CFE	P_m (N)
T08D3	7412.15	247.55	0.699	5179.04
T08D4	7387.11	250.93	0.696	5144.80
T08D5	7284.29	253.86	0.701	5107.20
T08D6	7177.01	256.63	0.707	5077.14
T10D3	10985.8	306.10	0.708	7777.97
T10D4	11080.6	310.85	0.699	7753.83
T10D5	10549.6	312.40	0.725	7647.03
T10D6	10478.3	313.14	0.719	7534.26
T12D3	15749.9	358.39	0.683	10756.63
T12D4	14592.6	364.84	0.733	10699.49
T12D5	13850.8	364.98	0.758	10496.18
T12D6	13776.6	365.07	0.752	10354.17

This level of agreement may be associated with the structural characteristics of the DHDW configuration, where the cross-section remains effectively symmetric and the folding process develops in a relatively uniform manner. Under such conditions, plastic hinges tend to form at comparable locations, leading to a more balanced deformation mode. In contrast to the SHDW configuration with an external plate, the symmetric arrangement of the double-hat sections reduces localized interaction effects.

Consequently, the load transfer and energy absorption mechanisms are more consistent with the assumptions adopted in the analytical formulation. The results therefore indicate that the predictive capability of the analytical model is closely related to how well the actual structural response reflects its underlying assumptions, particularly with respect to symmetry and idealized hinge formation.

This observation further highlights the sensitivity of analytical predictions to geometric symmetry and deformation characteristics. These differences in prediction accuracy are further illustrated in Fig. 17.

4.3. Analysis of simulation results

a) SHDW with changes in thickness and distance between two walls

Fig. 18 shows the deformation mode of the single-hat T10D4W20 column under bending-induced crushing at a rotation angle of 14° . The response evolves from global bending to progressive folding as the outer wall moves inward and contacts the inner wall, triggering deformation in both walls and forming multiple plastic hinge lines. This behavior is generally consistent with the bending collapse mode illustrated in Fig. 3 and is associated with enhanced energy absorption.

Table 11. Effect of spot weld pitches on the numerical crashworthiness performance of SHDW ($t = 1$ mm, $D = 4$ mm)

Specimen	P_{peak} (N)	SEA (J/kg)	CFE	P_m (N)
W15	11094.6	314.78	0.72	7980.75
W20	11084	317.25	0.72	8011.07
W25	11078.2	311.52	0.72	7940.12
W30	11003.3	315.43	0.73	7992.38

Based on Table 10, wall thickness t is the dominant parameter affecting the energy absorption capacity of SHDW structures at $W = 20$ mm. Increasing t from 0.8 mm to 1.2 mm significantly raises SEA from 247.55 J/kg to over 358.39 J/kg, along with corresponding increases in P_{peak} and P_m . For a given thickness, increasing the inner-wall distance D from 3 mm to 6 mm slightly reduces the peak force (e.g., from 15.75 kN to 13.78 kN in the T12 group) while maintaining high SEA .

The CFE remains consistently high (0.69–0.75), with the T12D6 configuration achieving the highest SEA (365.07 J/kg) and a CFE close to 0.75, indicating an optimal balance between energy absorption and crushing stability.

Fig. 19 compares the deformation patterns of the T12D6 and T08D6 configurations. Both exhibit similar symmetric progressive folding without noticeable asymmetric deformation. However, the T12D6 specimen shows more extended high-stress regions in the highlighted impact zones, whereas the T08D6 configuration presents more localized stress concentrations. Despite this difference, the overall deformation mechanisms remain comparable, consistent with their similar force–displacement responses.

The above trends are consistent with the force–displacement curves in Fig. 20 and the SEA results in Fig. 21. In all cases, the curves for $t = 1.2$ mm lie above those for thinner walls, indicating higher stiffness and mean crushing force, while SEA increases nearly linearly with thickness. The numerical force–displacement responses exhibit pronounced oscillations associated with successive fold formation during progressive collapse, leading to slightly lower P_m compared to the idealized analytical predictions

b) SHDW with changes in spot weld pitches

Table 11 shows that varying weld spacing from 15 to 30 mm has a negligible influence on the structural response, with both peak and mean forces changing by less than 1%. Energy absorption and SEA remain nearly unchanged, with a maximum at W20, while CFE shows a slight increase at larger spacing. Overall, the effect of weld spacing is minimal. These trends are also reflected in the force–displacement responses (Fig. 22) and the corresponding energy absorption indices (Fig. 23).

c) DHDW with changes in thickness and distance between two walls

In contrast to the SHDW configuration, the DHDW col-

umn exhibits stronger interaction between the two hat sections. As shown in Fig. 24, both hats deform simultaneously in a symmetric manner, similar to the general bending collapse mode. However, the deformation becomes more interactive, leading to a more extensive stress distribution and a larger number of plastic hinge lines across the section. This enhanced interaction stabilizes the folding process and results in improved energy absorption compared with SHDW.

Based on Table 12, for DHDW configurations with constant weld spacing ($W = 20$ mm), wall thickness has a pronounced effect on energy absorption performance. As t increases from 0.8 mm to 1.2 mm, SEA rises significantly by over 35%, while CFE remains consistently high (≈ 0.744) and slightly increases with thickness, indicating improved crushing stability.

In contrast, the influence of inner-wall distance D is relatively limited, with only moderate variations in SEA and minimal changes in CFE . Compared to SHDW, DHDW exhibits both higher SEA and consistently higher CFE , demonstrating a more efficient and stable energy absorption mechanism. Among all cases, DHDW–T12D6W20 achieves the best overall performance ($SEA \approx 480.83$ J/kg, $CFE \approx 0.7495$). To further illustrate the effect of thickness, the deformation patterns of DHDW–T08D6W20 and DHDW–T12D6W20 are compared based on FEM results.

Fig. 25 compares the deformation modes and stress distributions of the T12D6W20 and T08D6W20 configurations. Both configurations exhibit a similar overall deformation mode, consistent with the trend observed in the SHDW case. The primary difference lies in the stress distribution near the impact zones, where the thicker specimen (T12) shows more extended high-stress regions compared to the more localized concentrations in T08. This indicates a more distributed plastic deformation in the thicker configuration, consistent with the higher SEA reported in Table 12.

Figures 26 and 27 show that increasing thickness enhances both the crushing response and energy absorption of DHDW structures. The force–displacement curves exhibit a more stable plateau with reduced oscillations and a limited post-peak drop compared to SHDW, while SEA increases nearly linearly with consistently high CFE , confirming the dominant role of thickness.

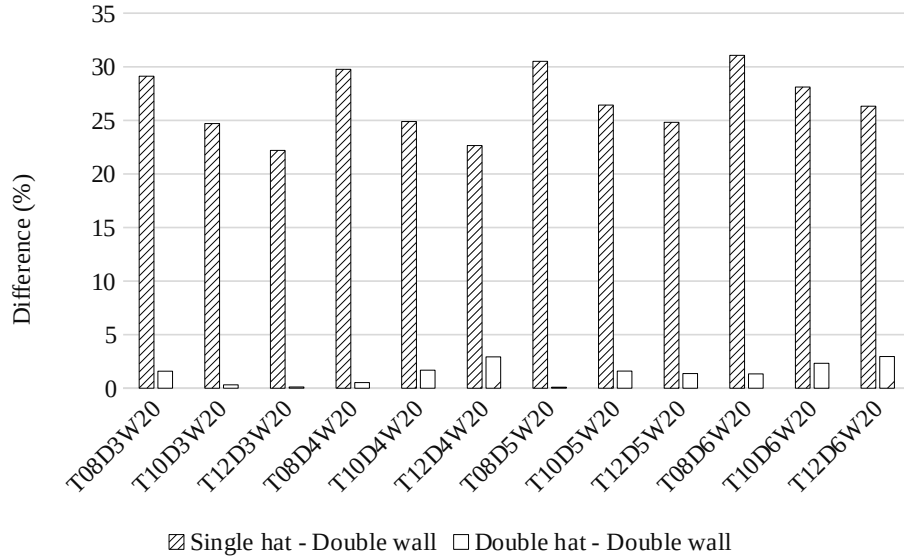


Figure 17. Comparison of percentage difference between theoretical and numerical results for single-hat and double-hat configuration

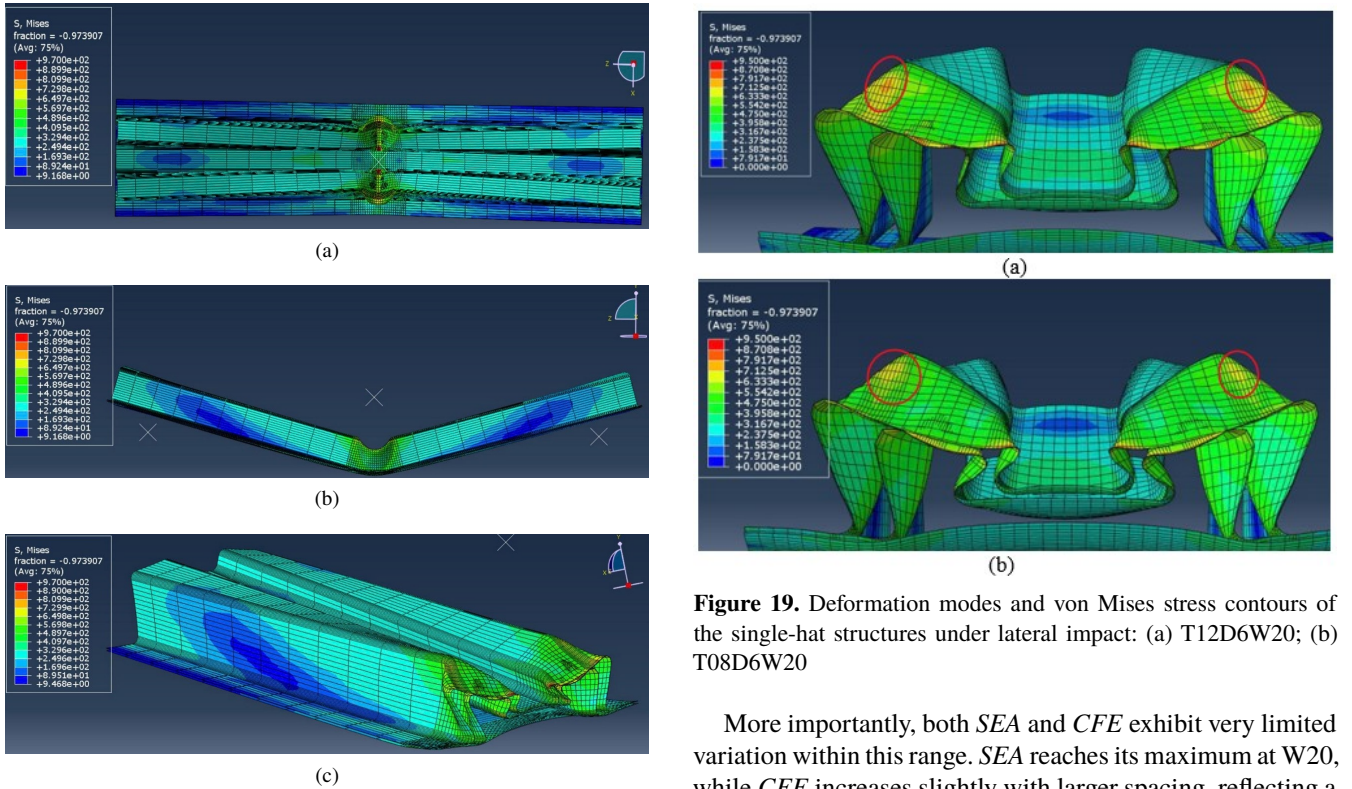


Figure 18. FEM results for bending behavior of SHDW-T10D4W20 at rotation angle $\theta=14$ degrees

d) DHDW with changes in spot weld pitches

Based on Table 13, for DHDW configurations with constant thickness ($t = 1.0$ mm) and inner-wall distance ($D = 4$ mm), varying the weld spacing from 15 mm to 30 mm has only a minor influence on structural performance, as also reflected in the force–displacement curves (Fig. 28) and energy absorption trends (Fig. 29). The force–displacement response remains largely unchanged, with only a slight reduction in peak force, indicating that the global stiffness and load-carrying capacity are well preserved.

Figure 19. Deformation modes and von Mises stress contours of the single-hat structures under lateral impact: (a) T12D6W20; (b) T08D6W20

More importantly, both *SEA* and *CFE* exhibit very limited variation within this range. *SEA* reaches its maximum at W20, while *CFE* increases slightly with larger spacing, reflecting a marginal improvement in crushing stability. The variation of both indicators remains within approximately 1–3%, confirming that weld spacing mainly fine-tunes the response without altering the overall crushing mechanism.

In addition, DHDW exhibits significantly lower sensitivity to weld spacing, as all configurations (W15–W30) deliver nearly equivalent *SEA* and *CFE* values, whereas SHDW shows a distinct optimal spacing. This highlights the superior robustness of the double-hat configuration under practical manufacturing conditions.

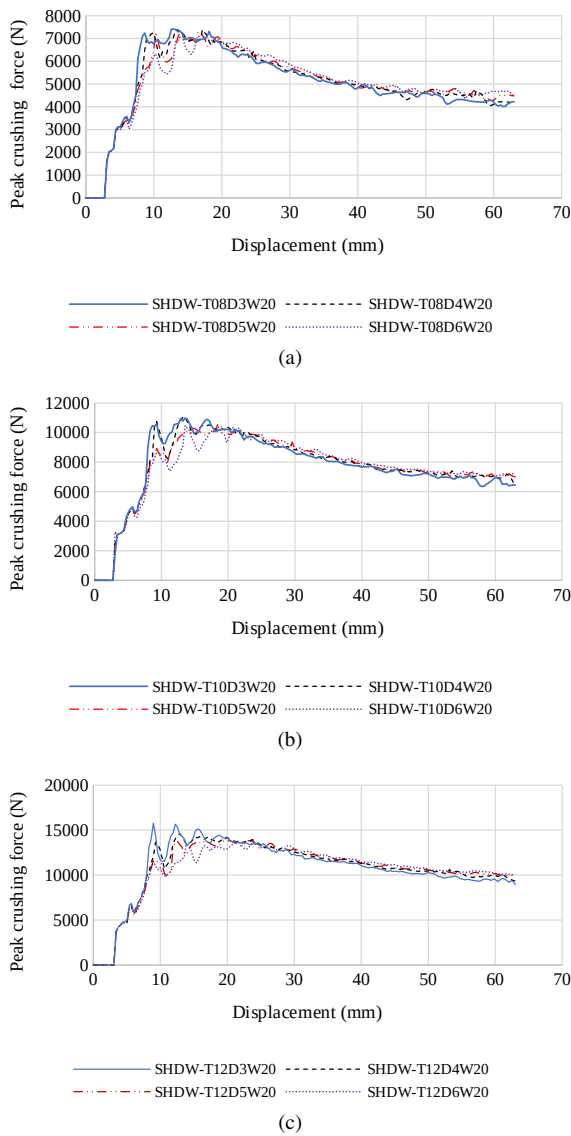


Figure 20. FEM results for bending crushing response of SHDW at different thickness and distances between inner and outer walls: (a) PCF vs. displacement of SHDW-T08; (b) PCF vs. displacement of SHDW-T10; (c) PCF vs. displacement of SHDW-T12

Overall, W20 provides the best balance in terms of *SEA*, while W30 offers slightly higher *CFE*. However, given the minimal differences, any configuration within this range can be considered effective, confirming the stability and reliability of the DHDW design.

e) Comparison between SHDW and DHDW

To clarify the structural advantages of the double-hat configuration, the deformation modes of SHDW and DHDW columns are compared based on FEM results. Two representative stages are considered: the onset of folding shortly after peak force and the progressive folding stage within the stable plateau. Fig. 30 shows the deformation patterns at the first stage (displacement = 24.63 mm).

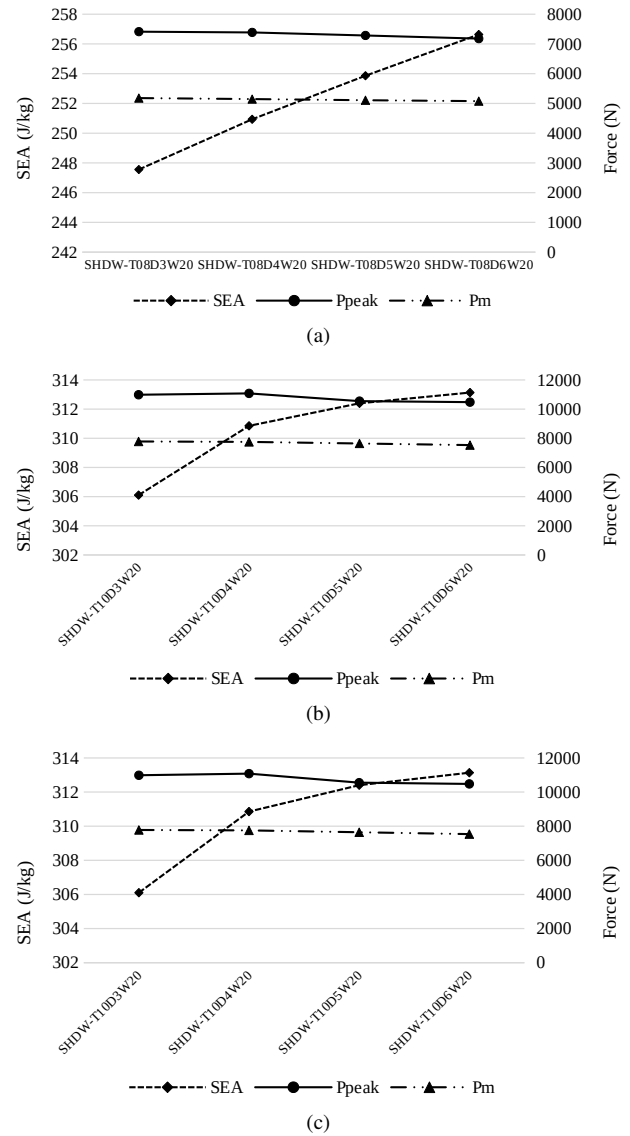


Figure 21. The relevant chart of energy absorption indices of SHDW at different thickness and distances between inner and outer walls: (a) PCF, MCF, SEA vs. displacement of SHDW-T08; (b) PCF, MCF, SEA vs. displacement of SHDW-T10; (c) PCF, MCF, SEA vs. displacement of SHDW-T12

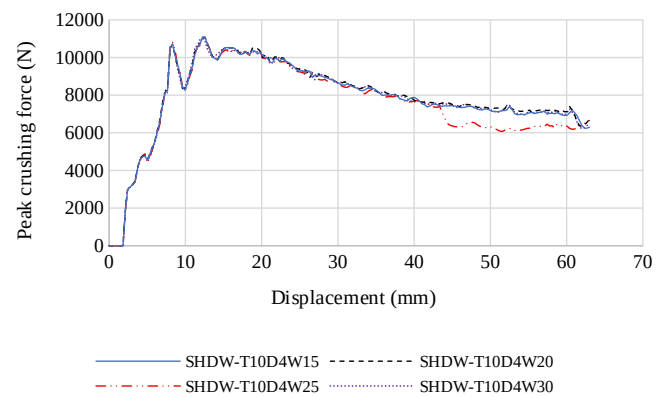


Figure 22. FEM results for bending crushing response of SHDW at different spot weld pitches

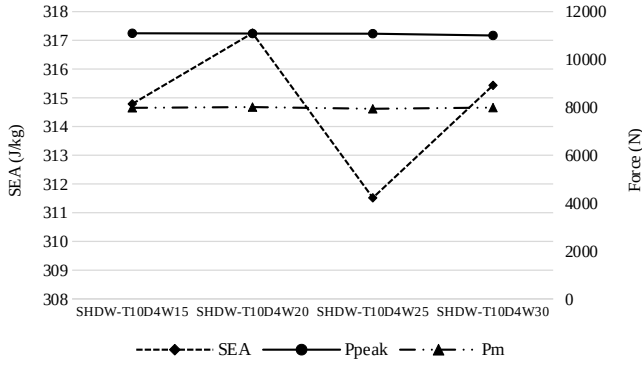


Figure 23. The relevant chart of energy absorption indices of SHDW at different spot weld pitches

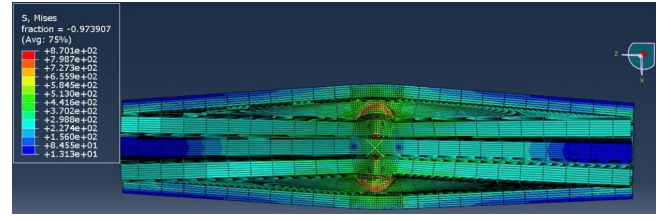
Table 12. Effect of wall thickness and inner-wall distance on the numerical crashworthiness performance of DHDW ($W = 20$ mm)

Specimen	P_{peak} (N)	SEA (J/kg)	CFE	P_m (N)
T08D3	14531.4	330.15	0.721	10477.40
T08D4	14591.0	338.89	0.724	10567.61
T08D5	14519.2	345.63	0.731	10610.96
T08D6	14721.8	354.99	0.728	10723.13
T10D3	20956.0	390.42	0.735	15391.63
T10D4	21416.5	404.80	0.732	15674.25
T10D5	21209.9	410.96	0.737	15627.48
T10D6	21301.3	419.10	0.738	15710.86
T12D3	27880.3	444.50	0.751	20946.79
T12D4	29095.3	465.37	0.739	21510.73
T12D5	28535.6	467.17	0.740	21127.06
T12D6	28586.4	480.83	0.749	21427.91

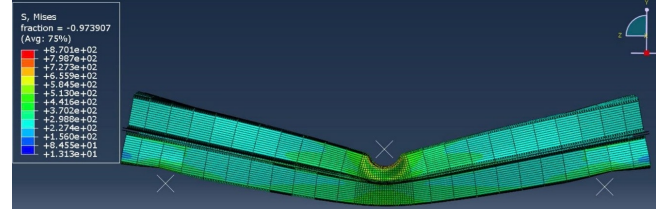
Table 13. Effect of spot weld pitches on the numerical crashworthiness performance of DHDW ($t = 1$ mm, $D = 4$ mm)

Specimen	P_{peak} (N)	SEA (J/kg)	CFE	P_m (N)
W15	21600.6	402.75	0.714	15424.72
W20	21416.5	404.80	0.732	15674.25
W25	21198.7	404.28	0.737	15629.51
W30	20782.9	398.10	0.741	15398.43

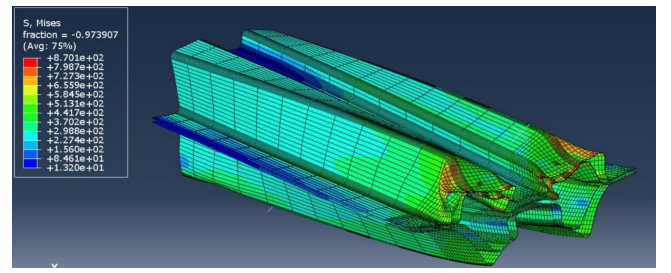
At a displacement of 24.63 mm, both configurations exhibit the onset of folding with similar interaction between the outer and inner walls. However, a clear difference appears in the supporting region (highlighted by the red-circled zone). In the SHDW column, the flat plate already undergoes noticeable upward bending, forming a curved profile, whereas the corresponding lower part of the DHDW configuration remains largely undeformed and retains its original geometry. This suggests that bending deformation is transferred to the supporting plate at an early stage in SHDW, while DHDW maintains a more stable structural support.



(a)

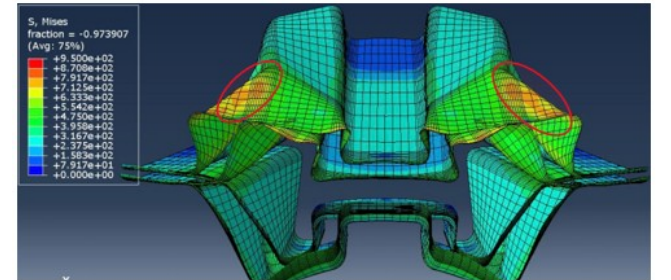


(b)

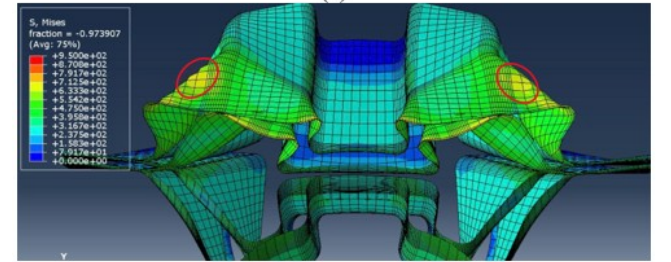


(c)

Figure 24. FEM results for bending behavior of DHDW-T10D4W20 at rotation angle $\theta = 14$ degrees



(a)



(b)

Figure 25. Deformation modes and von Mises stress contours of the double-hat structures under lateral impact: (a) T12D6W20; (b) T08D6W20

To further examine how this initial difference evolves into a stable crushing mechanism, the deformation patterns at a later stage within the progressive folding plateau are analyzed in the following section.

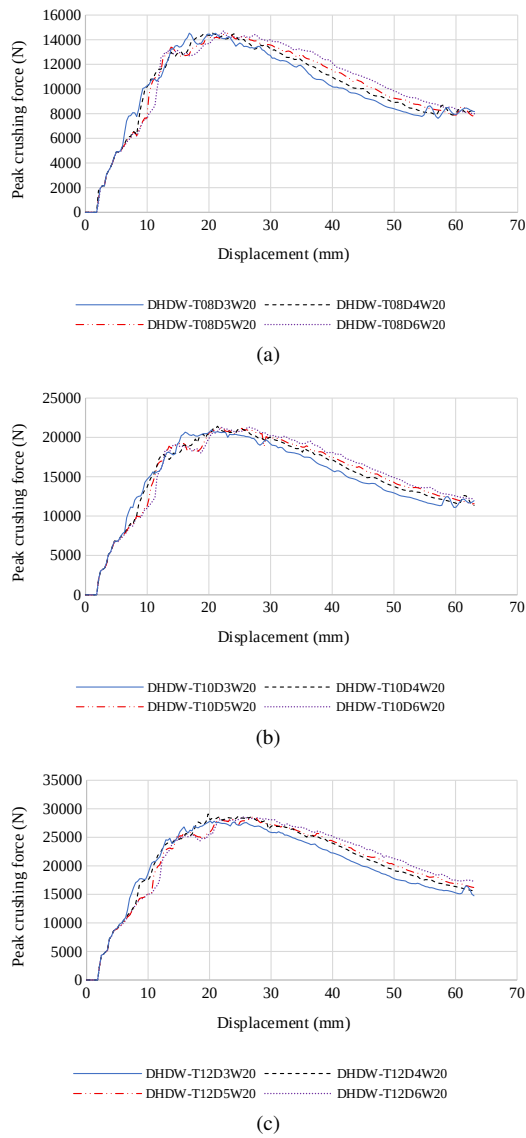


Figure 26. FEM results for bending crushing response of DHDW at different thickness and distances between inner and outer walls: (a) PCF vs. displacement of DHDW-T08; (b) PCF vs. displacement of DHDW-T10; (c) PCF vs. displacement of DHDW-T12

At the final stage of deformation (displacement = 60 mm), Fig. 31 shows that SHDW exhibits severe bending of the supporting plate with non-uniform folding, whereas DHDW retains a more symmetric and stable folding sequence. The double-hat configuration ensures continuous load transfer through both walls, resulting in a more uniform energy dissipation and preventing early loss of stability.

This difference is also reflected in the crashworthiness indicators. As shown in Tables 10 and 12, DHDW-T10D6 achieves a significantly higher SEA (419.10 J/kg vs. 313.14 J/kg) together with a slightly higher CFE (0.738 vs. 0.719) compared to SHDW-T10D6. This improvement is attributed to additional plastic hinge formation in the lower hat, which enhances energy absorption efficiency.

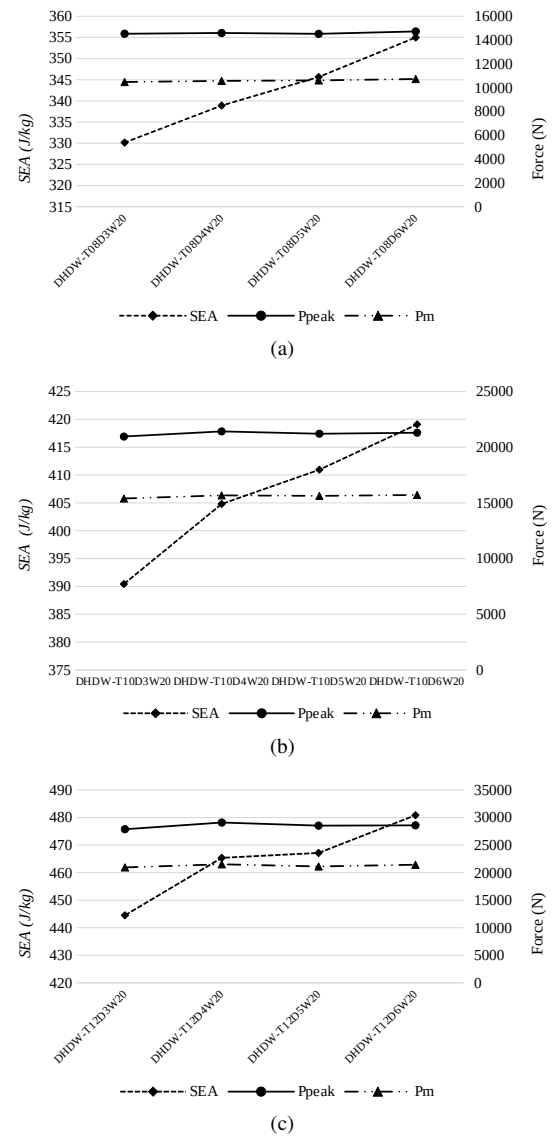


Figure 27. The relevant chart of energy absorption indices of DHDW at different thickness and distances between inner and outer walls: (a) PCF, MCF, SEA vs. displacement of DHDW-T08; (b) PCF, MCF, SEA vs. displacement of DHDW-T10; (c) PCF, MCF, SEA vs. displacement of DHDW-T12

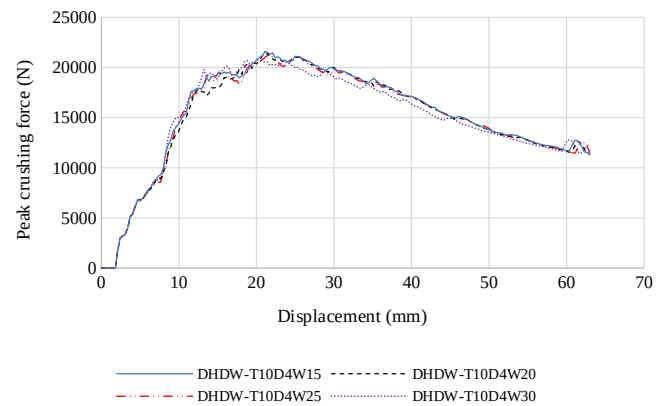


Figure 28. FEM results for bending crushing response of DHDW at different spot weld pitches

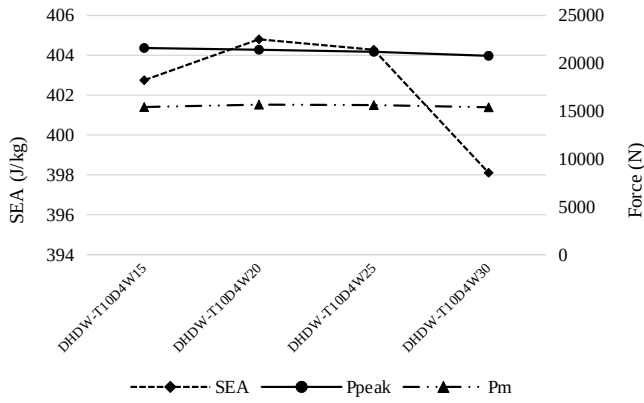


Figure 29. The relevant chart of energy absorption indices of DHDW at different spot weld pitches

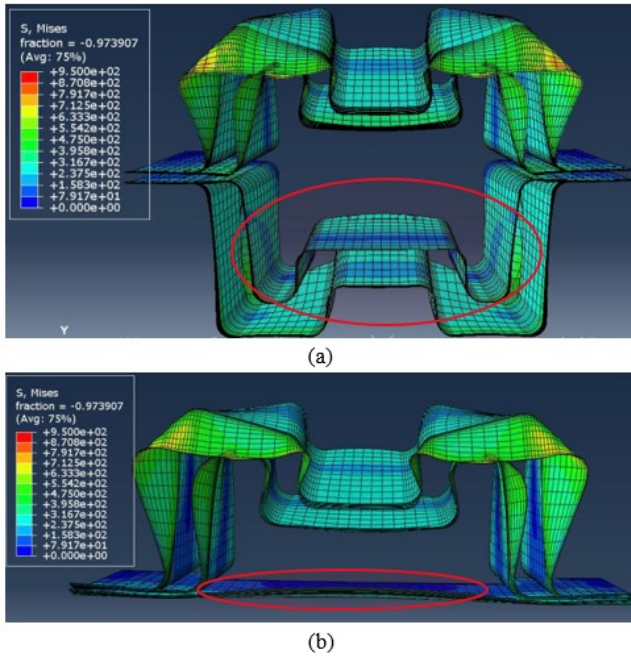
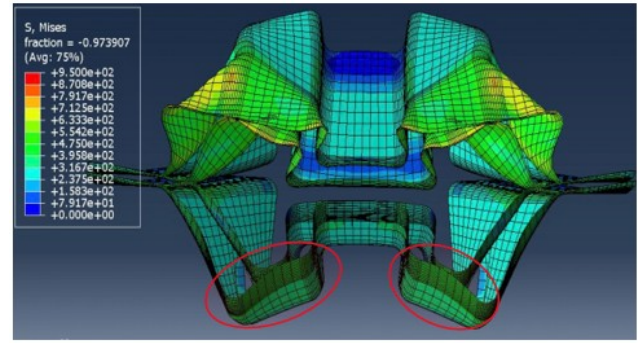
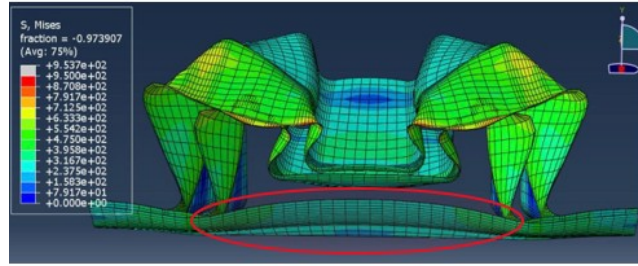


Figure 30. Comparison of deformation patterns between SHDW and DHDW columns at the onset of folding (displacement = 24.63 mm): (a) DHDW-T10D6W20; (b) SHDW-T10D6W20

The SEA-CFE comparison further confirms this trend. As illustrated in Fig. 32, all DHDW configurations consistently exhibit higher SEA values than their SHDW counterparts across the entire design space. In addition, the CFE of DHDW remains relatively stable within a narrow range, whereas SHDW shows more pronounced fluctuations. This demonstrates that the double-hat configuration enhances energy absorption efficiency while also improving the consistency of the crushing process, without sacrificing force stability.



(a)



(b)

Figure 31. Comparison of deformation patterns between SHDW and DHDW columns at the developed progressive folding stage within the stable crushing plateau (displacement = 60 mm): (a) DHDW-T10D6W20; (b) SHDW-T10D6W20

5. Conclusions

This study presented a combined theoretical and numerical investigation of thin double-walled multi-corner single-hat (SHDW) and double-hat (DHDW) columns under quasi-static lateral impact. The generalized super-beam formulation was validated against finite element results, showing that predictive accuracy strongly depends on geometric symmetry. While SHDW exhibited significant deviations (22–31%) due to asymmetric deformation, DHDW showed excellent agreement (0.09–2.96%), confirming the applicability of the analytical model to symmetric configurations.

Parametric analysis identified wall thickness as the dominant factor governing crashworthiness. Under identical conditions, DHDW achieved substantially higher SEA than SHDW while maintaining comparable or improved CFE, highlighting a more efficient and stable crushing mechanism. For instance, DHDW-T10D6 exhibited an SEA of 419.10 J/kg compared to 313.14 J/kg for SHDW-T10D6, along with a slightly higher CFE (0.738 vs. 0.719).

In contrast, inner-wall spacing and weld pitch had only minor effects, with performance variations remaining within a narrow range, indicating strong robustness against geometric and manufacturing variations.

Overall, DHDW consistently outperforms SHDW in energy absorption efficiency, crushing stability, and theoretical predictability, making it a promising lightweight solution for crashworthy structural design. Future work will focus on experimental validation under bending-dominated loading conditions.

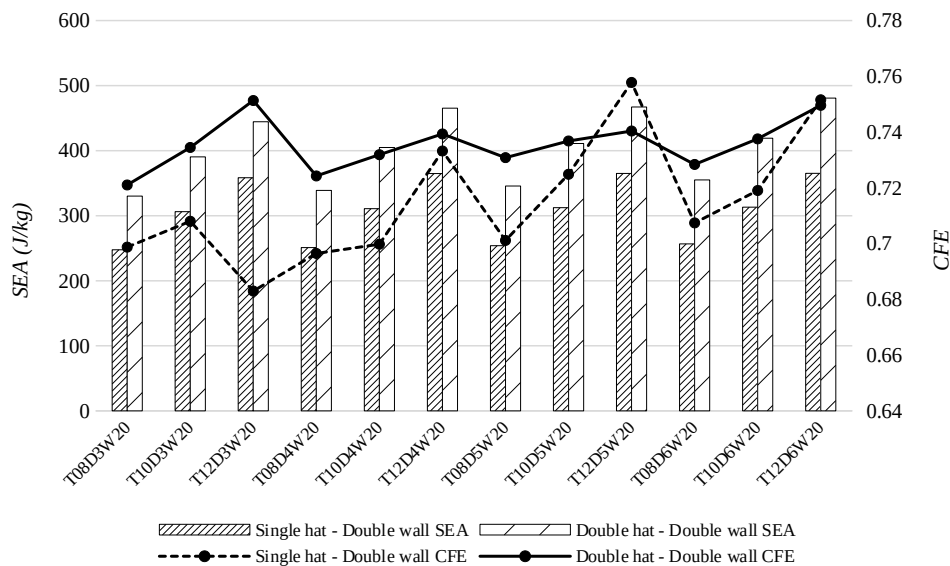


Figure 32. Comparison of energy absorption indices between SHDW and DHDW configurations

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